Section 10.3

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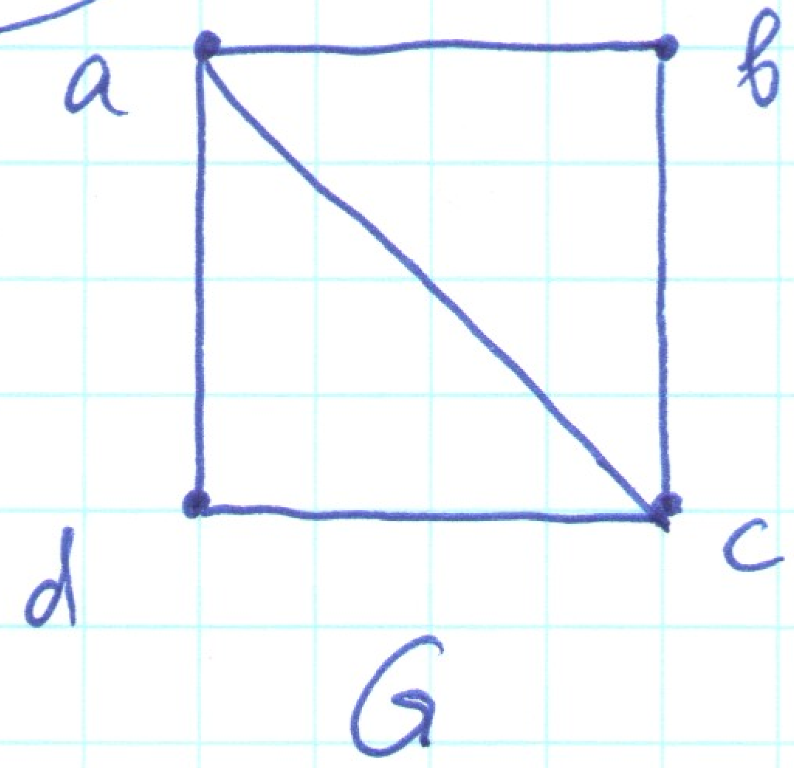

a → b | c | d

b → a | d | c

c → a | d

d → a | b | c

adjacency list of graph G

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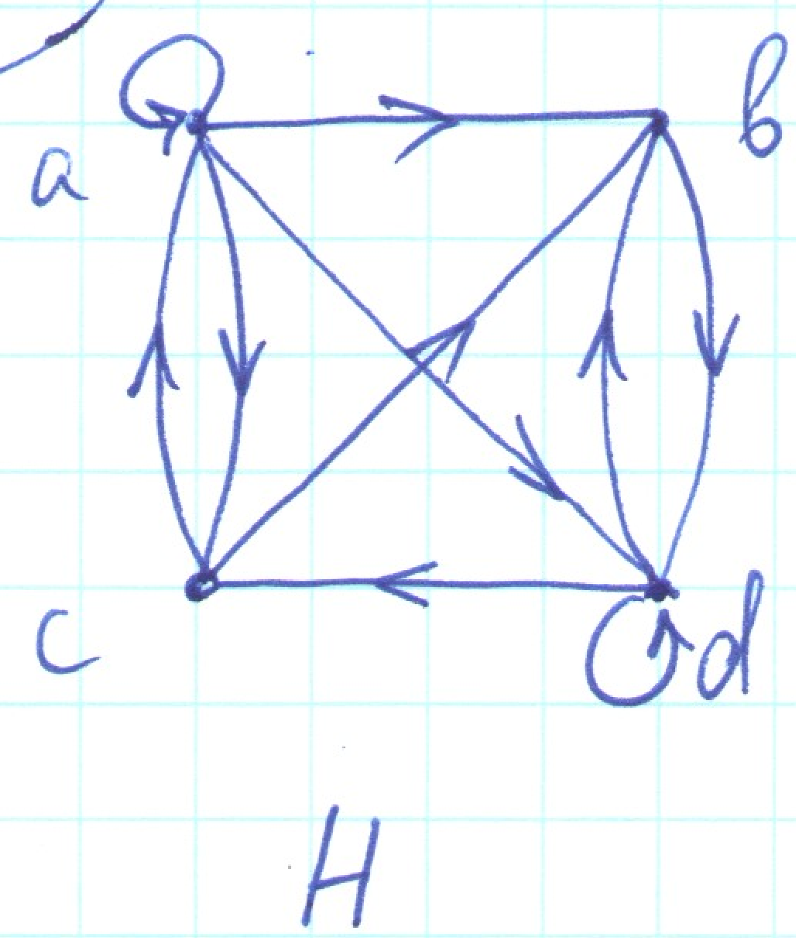

a → a | b | d | c

b → d

c → a | b

d → b | c | d

adjacency list of graph H

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$$
\begin{array}{c c}
 & \begin{array}{cccc} a & b & c & d \end{array} \\
\begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}
\end{array}
$$

adjacency matrix of graph H.

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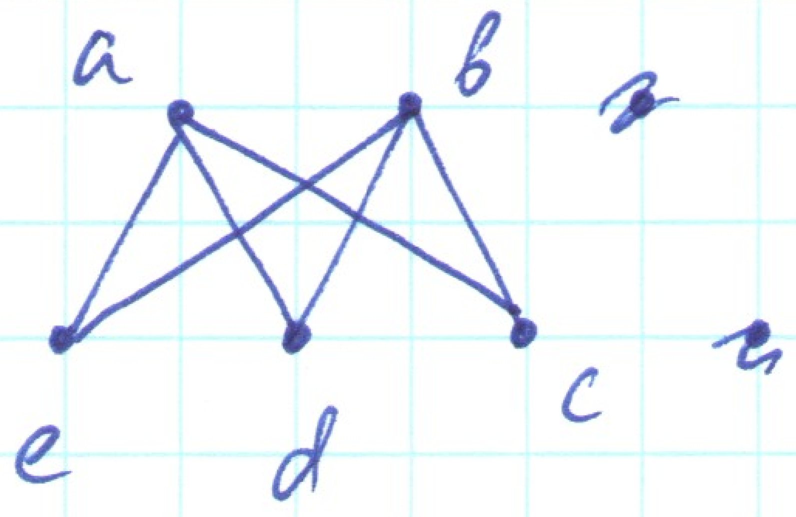
p.675/9(c)

$K_{2,3}$



$K_{2,3}$

a → c | d | e
b → c | d | e
c → a | b
d → a | b
e → a | b

adjacency list of $K_{2,3}$

$$\begin{array}{c|c} & a\ b\ c\ d\ e \\ \begin{array}{c} a\\ b\\ c\\ d\\ e \end{array} & \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

adjacency matrix of $K_{2,3}$

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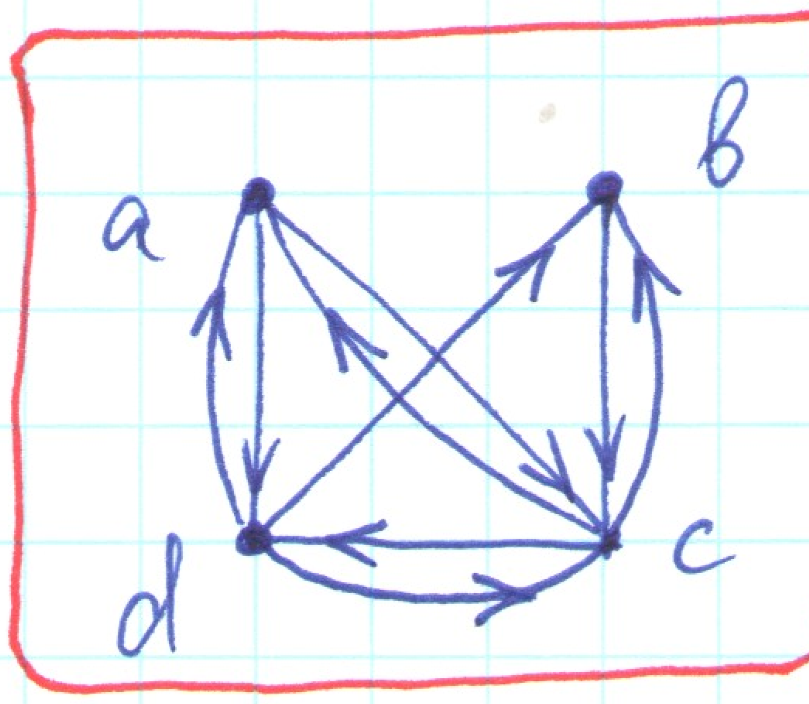
$$\begin{array}{c|c} & a\ b\ c\ d \\ \begin{array}{c} a\\ b\\ c\\ d \end{array} & \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$



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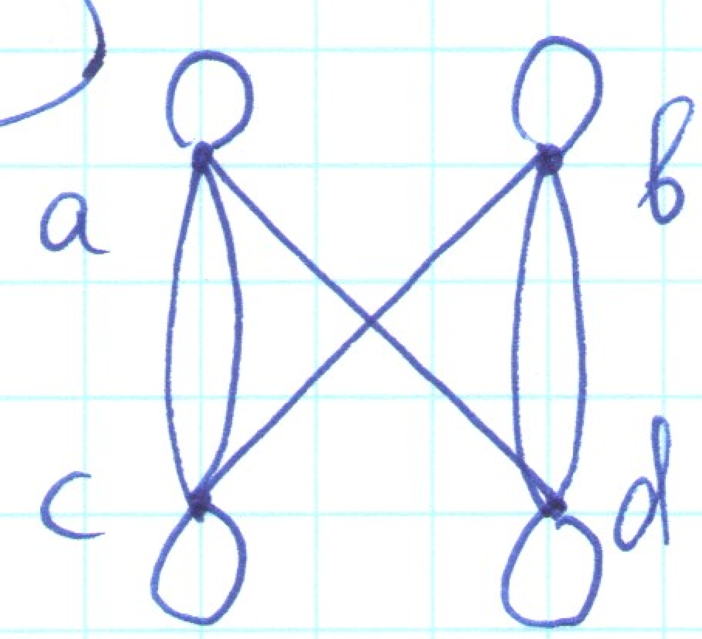

this is a pseudograph

$$\begin{array}{c|c} & a\ b\ c\ d \\ \begin{array}{c} a\\ b\\ c\\ d \end{array} & \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix} \end{array}$$

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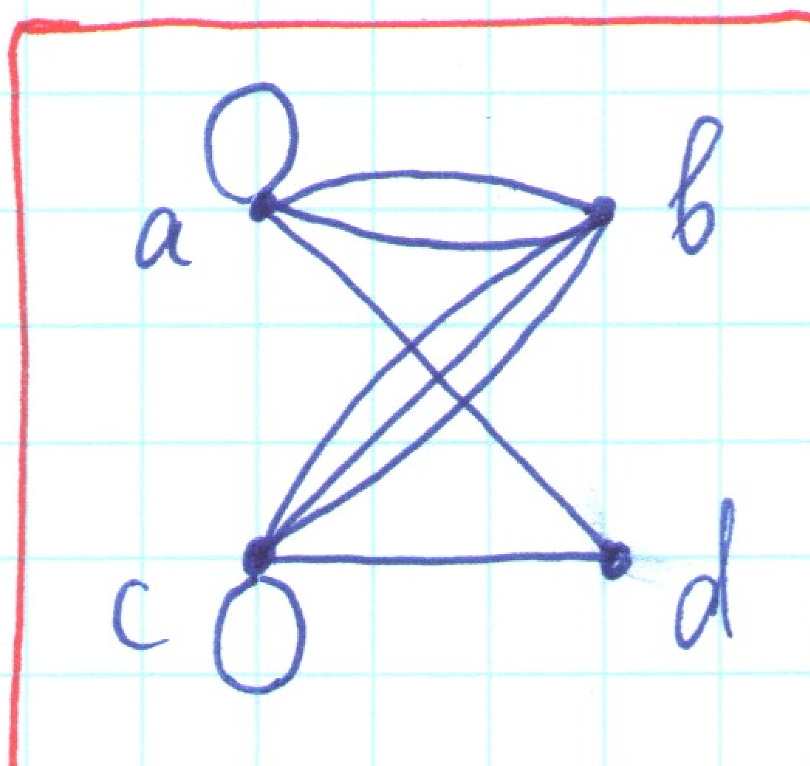
$$\begin{array}{c|c} & a\ b\ c\ d \\ \begin{array}{c} a\\ b\\ c\\ d \end{array} & \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$



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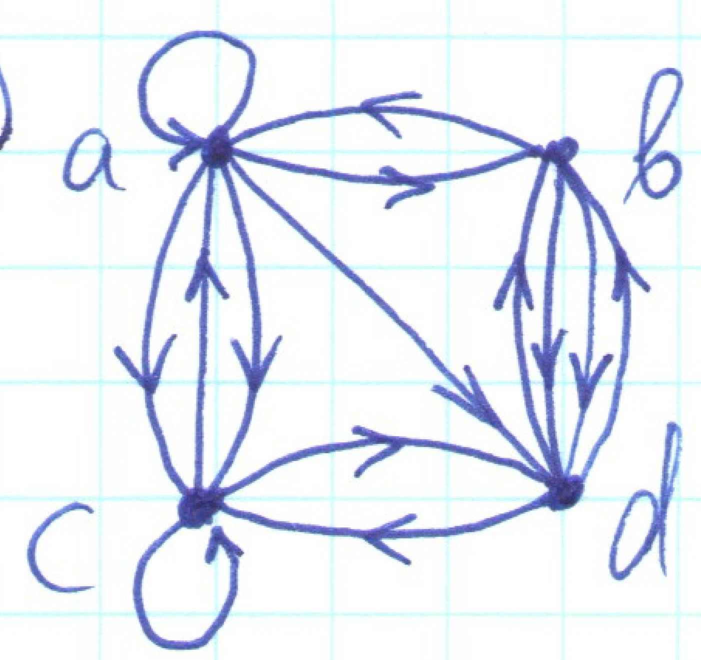

$$
\begin{array}{c|cccc}
 & a & b & c & d \\
\hline
a & 1 & 1 & 2 & 1 \\
b & 1 & 0 & 0 & 2 \\
c & 1 & 0 & 1 & 1 \\
d & 0 & 2 & 1 & 0
\end{array}
$$

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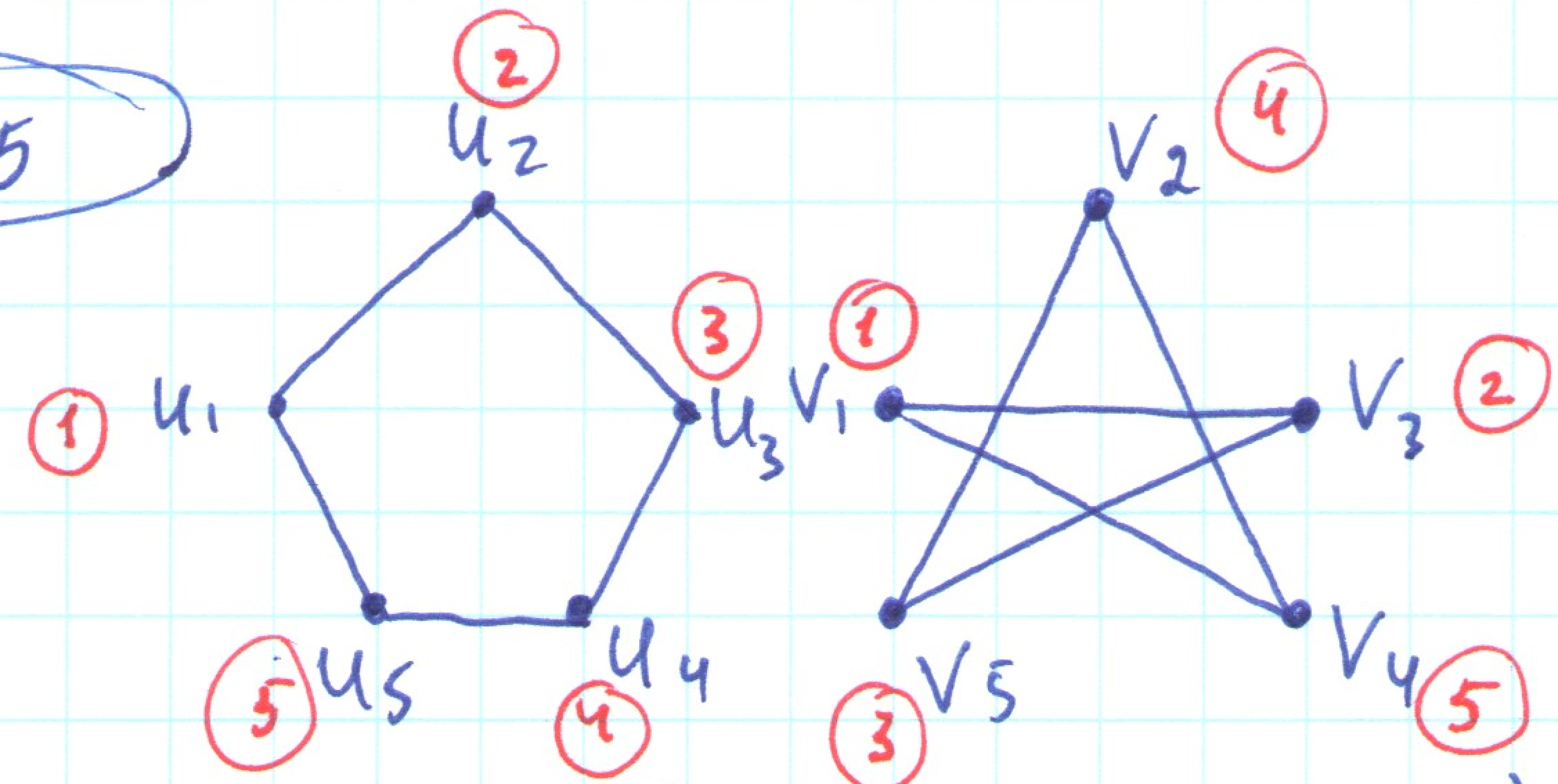

$G = (U, E_1)$ $H = (V, E_2)$

$|V| = 5$ $|U| = 5$
$|E_2| = 5$ $|E_1| = 5$

the degrees of all vertices in both graphs are $= 2$.

Let's begin with matching u_1 to v_1, followed by u_2 to v_3 (hence u_1 is adjacent to u_2 and v_1 is adjacent to v_3), then u_3 to v_5 (hence u_2 is adjacent to u_3 and v_3 is adjacent to v_5) - note that $(u_1, u_3) \notin G$ and $(v_1, v_5) \notin H$.

then let's map u_4 to v_2, followed by u_5 to v_4

We found a mapping $f : U \to V_1$, such that ← bijective function

$(u_1, u_2) \in E_1$ and $(v_1, v_3) \in E_2$,
$(u_2, u_3) \in E_1$ and $(v_3, v_5) \in E_2$, $(u_3, u_4) \in E_1$ and $(v_3, v_5) \in E_2$,
$(u_4, v_5) \in E_1$ and $(v_2, v_4) \in E_2$, $(u_5, u_1) \in E_1$ and $(v_4, v_1) \in E_2$.

Therefore, two graphs are isomorphic.

isomorphism → $f = \{ (u_1, v_1), (u_2, v_3), (u_3, v_5), (u_4, v_2), (u_5, v_4) \}$.

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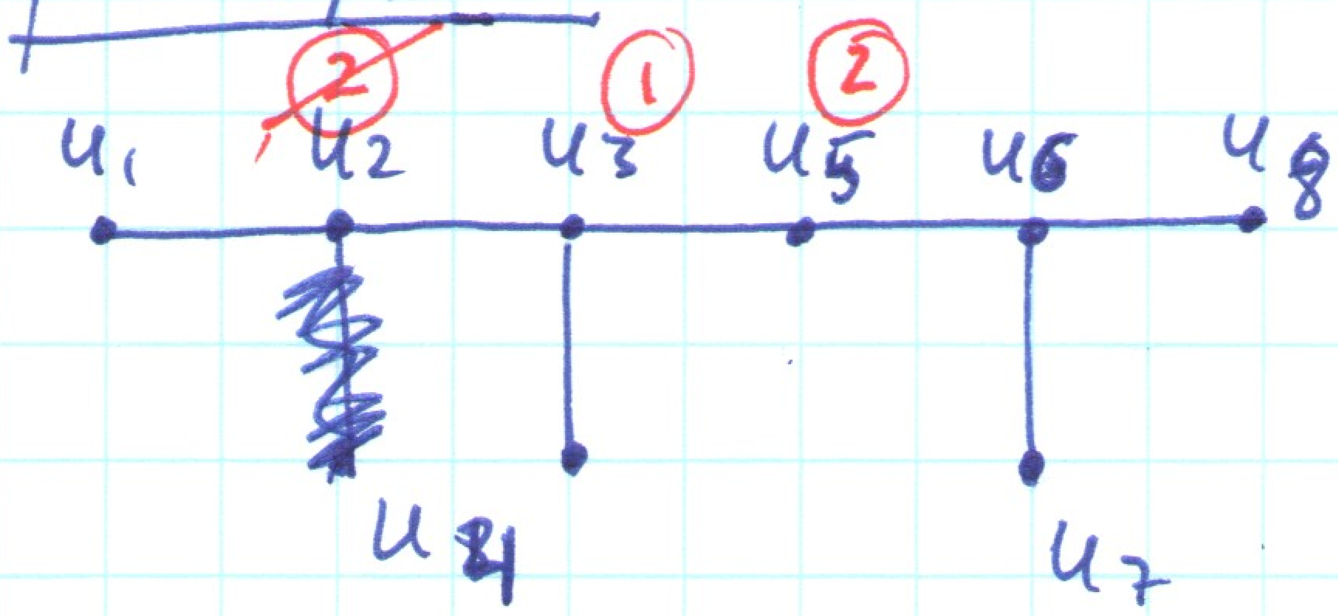
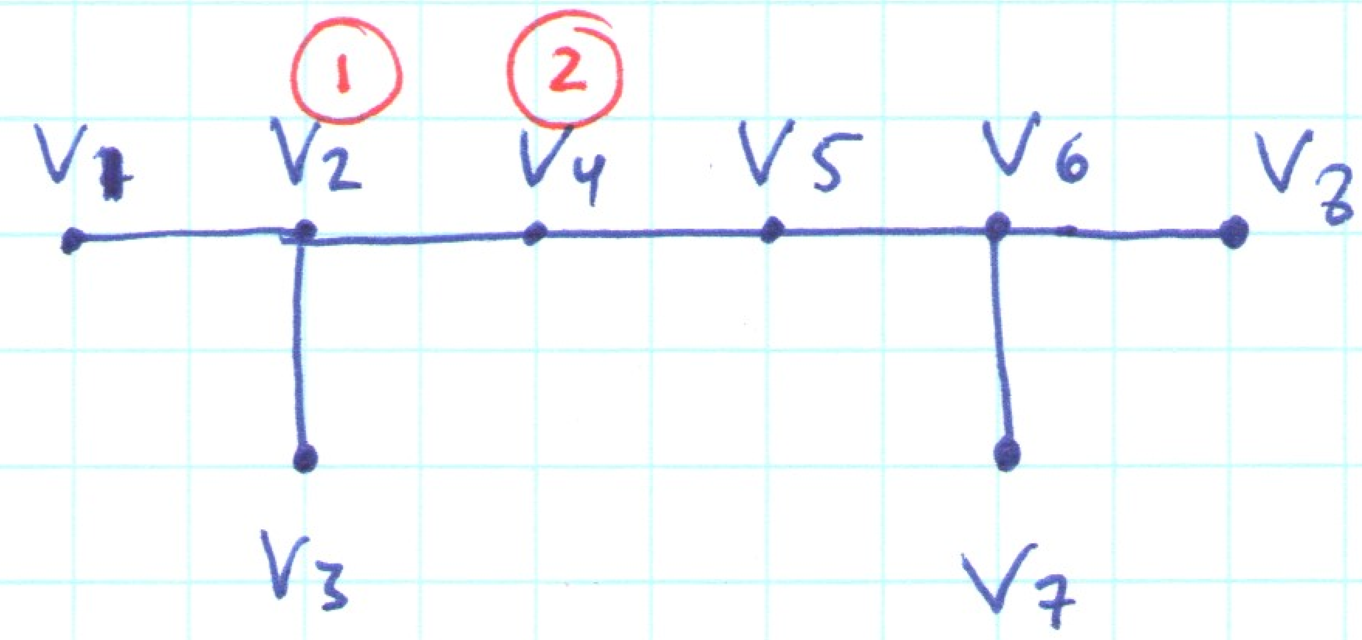


$G = (U, E_1)$



$H = (V, E_2)$

$|U| = 8$, $|E| = 7$ $\qquad$ $|V| = 8$, $|E| = 7$

$deg(u_3, u_6) = 3$, $deg(u_2, u_5) = 2$ $\qquad$ $deg(v_2, v_6) = 3$, $deg(v_4, v_5) = 2$

$deg(u_1, u_4, u_7, u_8) = 1$ $\qquad$ $deg(v_1, v_3, v_7, v_8) = 1$

Let's try to find a mapping from U to V so that it will satisfy "isomorphism" function requirement.

- $u_3 \to v_2$ $\quad$ note that u_3 is adjacent to u_2 and u_5 and $deg(u_2) = deg(u_5) = 2$, whereas v_2 is adjacent to only one vertex with degree 2, v_4, - here we will get a problem:

  $deg(u_3) = deg(v_2) = 3$

  if $u_2 \to v_4$

  $deg(u_2) = deg(v_4) = 2$

  then we won't be able to find a mapping for u_1, since it has degree 1, and v_4 is not adjacent to a vertex with degree 1.

  if $u_5 \to v_4$, then we cannot find mapping for u_6

  $deg(u_5) = deg(v_4) = 2$

  since $deg(u_6) = 3$, and v_4 is not adjacent to a vertex with degree 3.

- we are stuck.

Therefore graphs G u H are not isomorphic.

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p. 677/57 (a, b)

a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $|V_1| = |V_2| = 3$

$|E_1| = |E_2| = 4$

$G = (V_1, E_1)$ $H = (V_2, E_2)$

Let's work with adjacency matrix of H: (we will re-arrange order of vertices)

$$\begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \end{array}\begin{array}{c} \begin{array}{ccc} V_1 & V_2 & V_3 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array} \longrightarrow \begin{array}{c} \\ V_2 \\ V_3 \\ V_1 \end{array}\begin{array}{c} \begin{array}{ccc} V_2 & V_3 & V_1 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

now the adjacency matrix of H is equal to the adjacency matrix of G.

Therefore, the graphs represented by the adjacency matrices above are isomorphic.

b) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ $|V_1| = 4 = |V_2|$

$|E_1| = 8 \neq |E_2| = 10$

$G = (V_1, E_1)$ $H = (V_2, E_2)$

therefore the graphs represented by the given adjacency matrices are not isomorphic